MODELLING THE SHAPE OF OPEN CIRCUIT PULSES IN EDM

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Abstract: The shape of the gap voltage in open circuit conditions is calculated for an elementary pulse, during the rise time (t_r) and his subsequent time interval. Taking into consideration the inductive character of the discharge circuit and the capacitive character of the gap, such a pulse can generate damped voltage oscillations on the gap. This oscillating regime is of interest because it provides additional heat in the dielectric liquid, promoting the breakdown process [2]. The behaviour of the gap voltage is studied in three ways : by mathematic calculus, using a professional simulation software and by physical experiment. The conclusions are useful to estimate the contribution of voltage oscillations to the thermal activation of the dielectric liquid during the pre-ignition phase in the EDM process.

Keywords: EDM, open circuit, gap voltage, modelling

1. MODELLING THE EVOLUTION OF THE GAP VOLTAGE IN THE CASE OF AN ELEMENTARY PULSE, IN OPEN CIRCUIT CONDITIONS

During an open circuit pulse, the gap can be represented by a small capacitance C_o connected in parallel to a very large resistance R_o (MΩ), as shown in Fig. 1 [1]. The external circuit is represented by the current limitation resistance R, the parasite inductance L of the circuit, the static switch (a typical non linear element) and the start resistance R_a (several KΩ), assuring the minimal current to bring the static switch into conduction. The circuit is supplied by the DC ignition voltage U_a (in the range of 100V…300V).

Since $R_a \ll R_o$, the resistance of the parallel group $R_a || R_o$ can be aproximated to R_a.

The static switch is represented in this model by an ideal voltage source realising a complementary step, with linear variable edge (Fig.1).

This transient regime problem was solved using the *Laplace* method [3]. The signal $u_k(t)$, modelling the static switch, was decomposed in elementary components, admitting each a *Laplace* image. Further on, the problem has been solved using a superposition method.

The decomposition into simple components of $u_k(t)$ is shown in Fig.2. The $u_{k1}(t)$, $u_{k2}(t)$ components were successively applied to the circuit and the corresponding components $u_1(t)$, $u_2(t)$ of the gap voltage were calculated. Finally, the gap voltage was calculated by composition of these partial results.

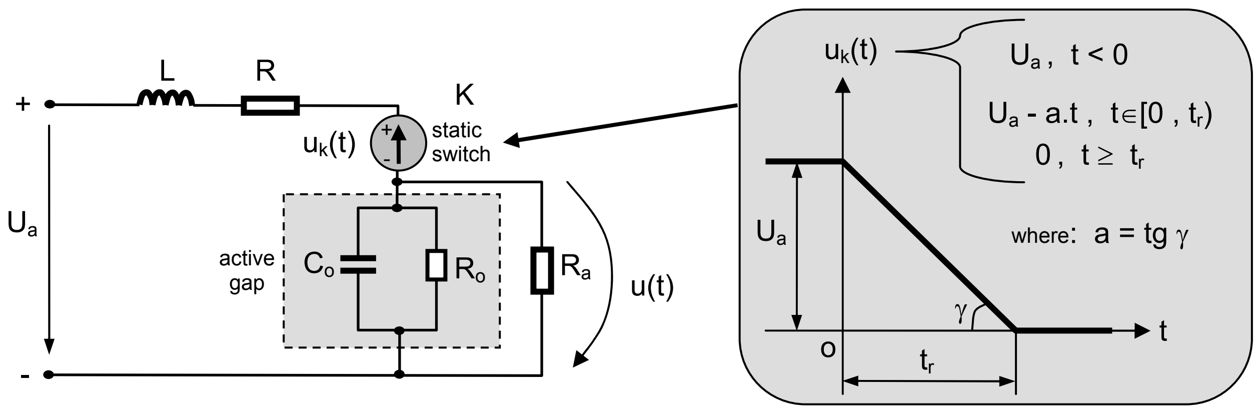

Fig.1. Model of the gap suplied by a linear rising voltage step, in open circuit conditions

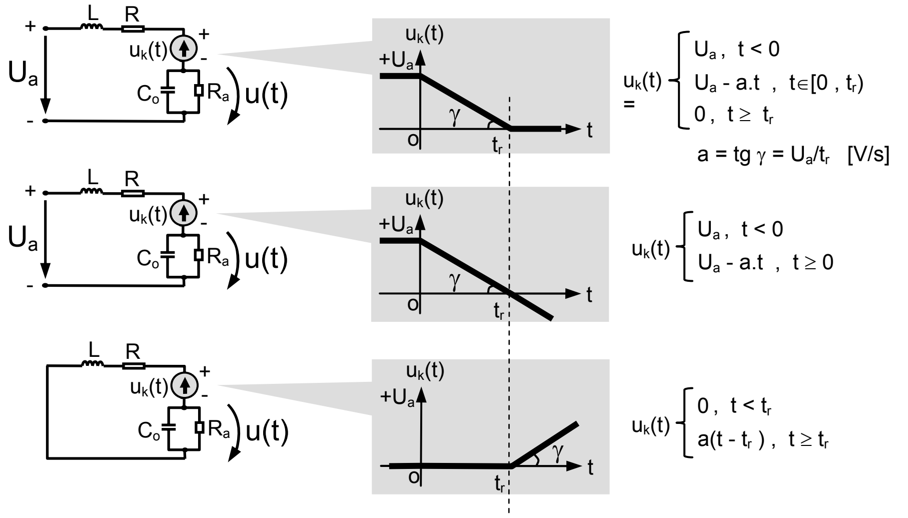

Fig.2. Decomposition of $u_k(t)$ into simple components

The Laplace images of the supply voltage U_a and of the $u_{k1}(t)$ component are [3] :

$$U_a(s)=\mathscr{L}\left[U_a\right]=\frac{U_a}{s} \qquad (1)$$

$$U_{k1}(s)=\mathscr{L}\left[u_{k1}(t)\right]=\frac{U_a}{s}-\frac{a}{s^2} \qquad (2)$$

A Laplace chard was associated to the circuit (Fig.3). The Laplace image $U_1(s)$ is :

$$U_1(s)=\frac{a}{LC_o}\cdot\frac{1}{s^2}\cdot\frac{1}{s^2+\left(\frac{R}{L}+\frac{1}{C_oR_a}\right)s+\frac{R+R_a}{LC_oR_a}} \qquad (3)$$

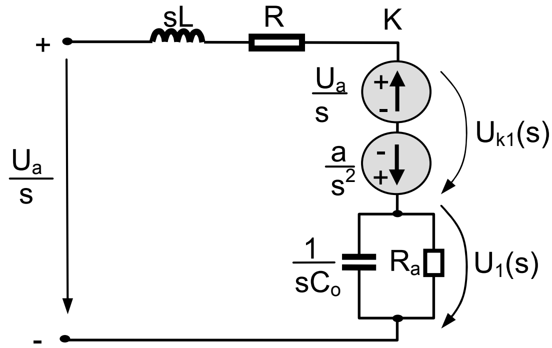

Fig.3. Laplace chard of the circuit in Fig.2.b

The discriminant of the second order expression of the denominator in (3) is :

$$\Delta=\left(\frac{R}{L}+\frac{1}{C_oR_a}\right)^2-4\cdot\frac{R+R_a}{R_a}\cdot\frac{1}{LC_o} \qquad (4)$$

To notice that in cases of practical interest, $\Delta < 0$.

(Example : for $R=10\Omega$, $R_a=2K\Omega$, $L=500\mu H$, $C_o=500pF$, the discriminant is $\Delta \approx -16.10^{12}s^{-2}$)

Relation (4) becomes :

$$U_1(s)=\frac{a}{LC_o}\cdot\frac{1}{s^2(s+\alpha)(s+\beta)} \qquad (5)$$

where α and β are complex conjugated constants of the type :

$$\alpha=\sigma-j\omega \quad \text{and} \quad \beta=\sigma+j\omega \qquad (6)$$

where :

$$\sigma=\frac{1}{2}\left(\frac{R}{L}+\frac{1}{C_oR_a}\right) \qquad (7)$$

$$\omega=\frac{1}{2}\sqrt{4\cdot\frac{R+R_a}{R_a}\cdot\frac{1}{LC_o}-\left(\frac{R}{L}+\frac{1}{C_oR_a}\right)^2} \qquad (8)$$

The component $u_1(t)$ results as the original function of (5) :

$$u_1(t)=\frac{a}{LC_o}\cdot\left(-\frac{\alpha+\beta}{\alpha^2\beta^2}+\frac{1}{\alpha\beta}\cdot t\right)+ \qquad (9)$$

$$+\frac{a}{LC_o}\cdot\left(\frac{1}{\alpha-\beta}\cdot\frac{1}{\alpha^2\beta^2}\cdot\left(\alpha^2\cdot e^{-\alpha t}-\beta^2\cdot e^{-\beta t}\right)\right)$$

After simple transformations, using Euler's formulas and usual trigonometric relations, the expression of the component $u_1(t)$ becomes : (10)

$$u_1(t)=-\frac{U_a}{t_r}\cdot\frac{2\sigma\omega_o^2}{\left(\sigma^2+\omega^2\right)^2}+\frac{U_a}{t_r}\cdot\frac{\omega_o^2}{\sigma^2+\omega^2}\cdot t+$$

$$+\frac{U_a}{t_r}\cdot\frac{\omega_o^2\left(\sigma^2-\omega^2\right)}{\omega\left(\sigma^2+\omega^2\right)}\cdot\frac{1}{\sqrt{1+4\left(\frac{\sigma\omega}{\sigma^2-\omega^2}\right)^2}}\cdot e^{-\sigma t}\cdot\sin(\omega t+\varphi)$$

where $\omega_o^2 = 1/LC_o$ is the angular resonant frequency of the circuit.

By a similar calculus, the second component of the gap voltage is : (11)

$$u_2(t)=\frac{U_a}{t_r}\cdot\frac{2\sigma\omega_o^2}{(\sigma^2+\omega^2)^2}-\frac{U_a}{t_r}\cdot\frac{\omega_o^2}{\sigma^2+\omega^2}\cdot(t-t_r)-$$

$$-\frac{U_a}{t_r}\cdot\frac{\omega_o^2(\sigma^2-\omega^2)}{\omega(\sigma^2+\omega^2)}\cdot\frac{1}{\sqrt{1+4\left(\frac{\sigma\omega}{\sigma^2-\omega^2}\right)^2}}\cdot e^{-\sigma(t-t_r)}\cdot\sin[\omega(t-t_r)+\varphi]$$

The superposition of the components $u_1(t)$, $u_2(t)$ has been realised as follows :

$$u(t)=\begin{cases}0\ , & t<0\\ u_1(t)\ , & t\in[\,0\ ,\ t_r\,)\\ u_1(t)+u_2(t)\ , & t\geq t_r\end{cases}\qquad(12)$$

Applying some usual transformations, the final result becomes :

- for $t < 0$: *u(t) = 0* (13)
- for $t \in [\,0\ ,\ t_r\,)$:

$$u(t)=-\frac{U_a}{t_r}\cdot\frac{2\sigma}{\omega_o^2}\cdot\left(\frac{R_a}{R_a+R}\right)^2+\frac{U_a}{t_r}\cdot\frac{R_a}{R_a+R}\cdot t+$$

$$+\frac{U_a}{t_r}\cdot\left(\frac{R_a}{R_a+R}\right)^3\cdot\frac{(\sigma^2-\omega^2)^2}{\omega\cdot\omega_o^4}\cdot e^{-\sigma t}\cdot\sin(\omega t+\varphi)$$

- for $t \geq t_r$:

$$u(t)=U_a\cdot\frac{R_a}{R_a+R}+A_r\cdot e^{-\sigma t}\cdot\sin(\omega t+\theta_r)$$

where : (14)

$$A_r=\frac{U_a}{t_r}\cdot\left(\frac{R_a}{R_a+R}\right)^3\cdot\frac{(\sigma^2-\omega^2)^2}{\omega\cdot\omega_o^4}\cdot\frac{1-e^{\sigma t_r}\cos\omega t_r}{\sqrt{1+\left(\frac{e^{\sigma t_r}\sin\omega t_r}{1-e^{\sigma t_r}\cos\omega t_r}\right)^2}}$$

$$\sigma=\frac{1}{2}\left(\frac{R}{L}+\frac{1}{C_oR_a}\right)\qquad\omega_o=\frac{1}{\sqrt{LC_o}}$$

$$\omega=\frac{1}{2}\sqrt{4\frac{R_a+R}{R_a}\cdot\frac{1}{C_oL}-\left(\frac{R}{L}+\frac{1}{C_oR_a}\right)^2}$$

$$\varphi=arctg\ \frac{2\sigma\omega}{\sigma^2-\omega^2}$$

$$\theta_r=arctg\ \frac{2\sigma\omega}{\sigma^2-\omega^2}+arctg\ \frac{e^{\sigma t_r}\sin\omega t_r}{1-e^{\sigma t_r}\cos\omega t_r}$$

It comes out that on the rising edge of a open circuit pulse ($t \in [\,0\ ,\ t_r\,)$) the gap voltage has a constant component, a linear rising component, and a damped harmonic component. On the beginning of the pulse porch the voltage has a constant component and a damped harmonic component.

Numeric calculus has been performed to evaluate the weigt of each of these components in the gap voltage u(t). *Considering a cylindric electrode Φ=40mm, a gap of d=20μm, diesel oil with a relative permitivity of $\varepsilon_r \approx 2$ as dyelectric liquid and the universal dyelectric constant ε_0=8,85419.10^{-11} F/m, the capacitance of the gap is $C_o=\varepsilon_0\varepsilon_r\pi\Phi^2/4d \approx 10^{-8}$ F = 10nF. The ignition voltage has been considered U_a = 200 V, and the rising time of the voltage pulse t_r = 8μs. The exterior circuit parameters have been considered as follows : R=30Ω, R_a=2KΩ, L=300μH. With these values, the constants in the expression of u(t) are :*
σ = 0,75.10^5 s^{-1}, ω_0= 5,77. 10^5 s^{-1},
ω = 5,76. 10^5 s^{-1}, φ = -0,259 rad = -14,84^o,
θ_r= -0,436 rad=-25^o, A_r = -32 V.

In this particular case, the u(t) voltage is :

- for $t < 0$: u(t) = 0 (15)
- for $t \in [\,0\ ,\ t_r\,)$:

$$u(t)=-10{,}94+24625000\cdot t+$$

$$+39{,}82\cdot e^{-0{,}75\cdot10^5 t}\cdot\sin(5{,}76\cdot10^5 t-0{,}259)\quad[V]$$

- for $t \geq t_r$:

$$u(t)=197-32\cdot e^{-0{,}75\cdot10^5 t}\cdot\sin(5{,}76\cdot10^5 t-0{,}436)\ [V]$$

It comes out that the linear component *24625000.t* in (15) is clearly dominant if comparatively with the other components of the gap voltage. In the [0, t_r) time interval, u(t) can be well approximated with a linear rising function.

As a conclusion, on the rising edge the gap voltage can be aproximated as follows:

- for $t < 0$: *u(t) = 0* (16)
- for $t \in [\,0\ ,\ t_r\,)$: $u(t)\approx\frac{U_a}{t_r}\cdot\frac{R_a}{R_a+R}\cdot t$
- for $t \geq t_r$:

$$u(t)=U_a\cdot\frac{R_a}{R_a+R}+A_r\cdot e^{-\sigma t}\cdot\sin(\omega t+\theta_r)$$

where A_r, σ, ω_0, ω, and θ_r are expressed in (14).

The waveform of u(t) is shown in Fig.4.

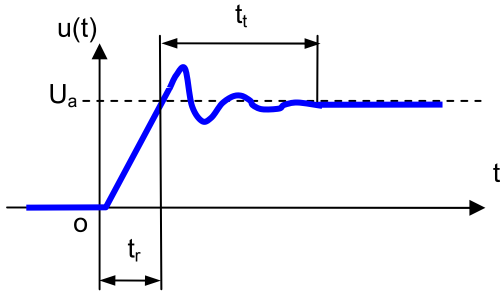

Fig.4. Waveform of the gap voltage u(t) on rising edge, in open circuit conditions

The duration of the transient regime (the transient time t_t) is depending on the damping constant σ [3] and can be approximated by:

$$t_t\approx3\cdot\frac{1}{\sigma}=\frac{3}{\frac{R}{L}+\frac{1}{C_oR_a}}\qquad(17)$$

2. VERIFYING THE RESULT USING A SIMULATION SOFTWARE

These theoretical results have been verified using the *Tina-Design Suite-v7* simulation software [4].

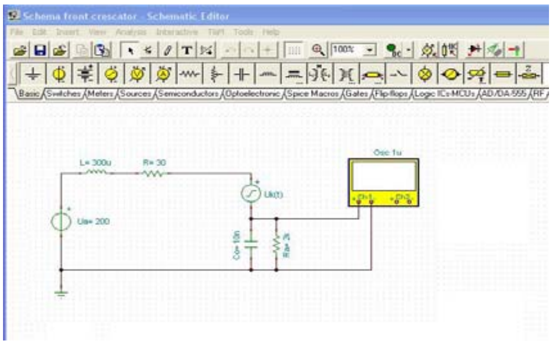

Fig.5. Structure of the simulated circuit

The structure of the virtual circuit used for simulaton is shown in Fig.5.

The simulation diagram is shown in Fig.6. The results prove that the mathematic model previously presented is qualitatively correct.

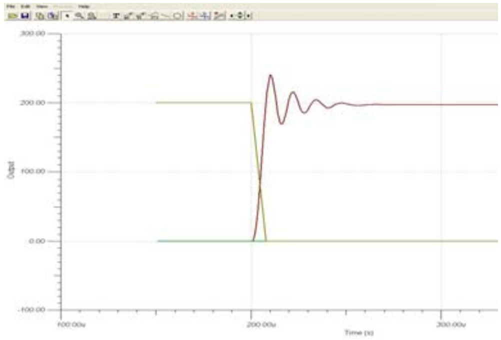

Fig.6. Simulation diagram

3. VERIFYING THE RESULT BY PHYSICAL EXPERIMENT

The experimental test was realized using two copper electrodes separated by a 20 μm gap, immersed in diesel oil.

The circuit was supplied with 120V/1KHz voltage pulses realized by a static switch. The oscillogram in Fig.7 shows the existence of small damped osccillations on the rising edge of the pulse.

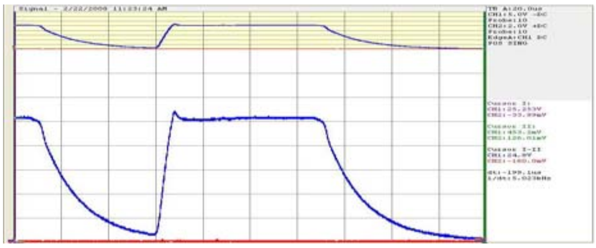

Fig.7. Oscillogram of the gap voltage in open circuit.

4. CONCLUSIONS

Open circuit pulses can generate damped voltage oscillations on the gap. This transient regime is of interest because it provides additional heat in the dielectric liquid. As a result, the probability to realise an electric discharge is higher.

The transient time t_t increases if the circuit inductance L is high, the limiting resistance R is low (that means the discharge current is high), the start resistance R_a is high and the equivalent capacitance of the gap C_o is high. The capacitance C_o is directly proportional to the active area of the tool-electrode and conversely proportional to the size of the gap. The duration and the intensity of the described transient regime are therefore dependent on the geometry of the gap.

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